Eur. Phys. J. C 29, 441–446 (2003)
Digital Object Identifier (DOI) 10.1140/epjc/s2003-01207-4

THE EUROPEAN PHYSICAL JOURNAL C

The $Z \to \gamma\gamma$, gg decays in the non-commutative standard model

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Received: 2 July 2002 / Revised version: 28 February 2003 /
Published online: 18 June 2003 –

Abstract. On non-commutative spacetime, the standard model (SM) allows new, usually SM forbidden, triple gauge boson interactions to occur. In this letter we propose the SM strictly forbidden $Z \to \gamma\gamma$ and $Z \to gg$ decay modes coming from the gauge sector of the non-commutative standard model (NCSM) as a place where non-commutativity could be experimentally discovered.

In this article we consider strictly SM forbidden decays coming from the gauge sector of the NCSM which could be probed in high energy collider experiments. This sector is particularly interesting from the theoretical point of view. It is the place where different models show the greatest differences. In particular there are models that do not require any new triple gauge boson interactions. This depends on the choice of representation. It is, however, important to emphasize that generically one should expect triple boson interactions. We will in particular argue that a model that does have new triple gauge boson interactions is natural as an effective theory of non-commutativity. Our main results are summarized in (17)–(19).

The idea that coordinates may not commute can be traced back to Heisenberg. A simple way to introduce a non-commutative structure into spacetime is to promote the usual spacetime coordinates x to non-commutative (NC) coordinates $\hat{x}$ with [1]

$$[\hat{x}^\mu, \hat{x}^\nu] = \mathrm{i}\theta^{\mu\nu}, \quad [\theta^{\mu\nu}, \hat{x}^\rho] = 0, \tag{1}$$

were $\theta^{\mu\nu}$ is a constant, real, antisymmetric matrix. The non-commutativity scale Λ_{NC} is fixed by choosing dimensionless matrix elements $c^{\mu\nu} = \Lambda_{\mathrm{NC}}^2\, \theta^{\mu\nu}$ of order one. The original motivation to study such a scenario was the hope that the introduction of a fundamental scale could deal with the infinities of quantum field theory in a natural way. The simple commutation relation (1) with constant $\theta^{\mu\nu}$ fails to provide a complete regularization [2], but more complicated non-commutative structures can indeed introduce spacetime lattice structures into the theory that are compatible with a deformation of *continuous* spacetime symmetries (see, e.g., [3]). This is in contrast to the situation in ordinary lattice field theory, where only discrete translation symmetries survive. Aside from these technical merits, the possibility of a non-commutative structure of spacetime is of interest in its own right and its experimental discovery would be a result of fundamental importance.

Non-commutative gauge theory has become a focus of interest in string theory and M-theory with the work given in [4]. Non-commutativity of spacetime is very natural in string theory and can be understood as an effect of the interplay of closed and open strings. The commutation relation (1) enters in string theory through the Moyal–Weyl star product

$$f \star g = \sum_{n=0}^{\infty} \frac{\theta^{\mu_1\nu_1} \ldots \theta^{\mu_n\nu_n}}{(-2\mathrm{i})^n n!} \partial_{\mu_1} \ldots \partial_{\mu_n} f \cdot \partial_{\nu_1} \ldots \partial_{\nu_n} g. \tag{2}$$

For the coordinates we have $x^\mu \star x^\nu - x^\nu \star x^\mu = \mathrm{i}\theta^{\mu\nu}$. The tensor $\theta^{\mu\nu}$ is determined by a NS $B^{\mu\nu}$-field and the open string metric $G^{\mu\nu}$ [5], which both depend on a given closed string background. The effective physics on D-branes is most naturally captured by non-commutative $U(N)$ gauge theory, but it can also be described by ordinary gauge theory. Both descriptions are related by the Seiberg–Witten (SW) map [6], which expresses non-commutative gauge

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fields in terms of fields with ordinary "commutative" gauge transformation properties.

Quantum field theory on non-commutative spacetime can be studied also independently of string theory. There are two major approaches. The original one based on actions that resembles that of Yang–Mills theory with matrix multiplication replaced by the Moyal–Weyl star product and a more recent one that utilizes the so-called Seiberg–Witten map to express non-commutative fields in terms of physical (commutative) fields. Both have their advantages and limitations. In the original approach unusual non-perturbative effects like UV/IR mixing [7] can be studied, but gauge theories are limited to the gauge group $U(N)$ in the fundamental representation. There are also indications of more fundamental problems in the rigorous definition of the S-matrix. The second approach treats non-commutativity strictly perturbatively via a Seiberg–Witten map expansion in terms of θ. A major advantage of the second approach is that models with any gauge group – including the one of the standard model – and any particle content can be constructed. Further problems that are solved in this approach include the charge quantization problem of NC Abelian gauge theories and the construction of covariant Yukawa couplings. The action is manifestly gauge invariant. It is written in terms of physical fields and their derivatives and should be understood as an effective model describing non-commutative effects in particle physics; see [8,9,17,10] and references therein.

Experimental signatures of non-commutativity have been discussed from the point of view of collider physics [11–14] as well as low energy non-accelerator experiments [14–16]. Two widely disparate sets of bounds on Λ_{NC} can be found in the literature: bounds of order 10^{11} GeV [15] or higher [14], and bounds of a few TeV from colliders [11–13]. All these limits rest on one or more of the following assumptions, which may have to be modified:
(1) θ is constant across distances that are very large compared with the NC scale;
(2) unrealistic gauge groups;
(3) non-commutativity down to low energy scales.

The decay of the Z-boson into two photons was previously considered in [14], where the authors rely on a non-commutative $U(1)$ model, i.e., not yet a bonafide non-commutative model of the electroweak sector or the standard model.

There are two essential points in which NC gauge theories differ from standard gauge theories. The first point is the breakdown of Lorentz invariance with respect to a fixed non-zero $\theta^{\mu\nu}$ background (which obviously fixes preferred directions) and the other is the appearance of new interactions (three-photon coupling, for example) and the modification of standard ones. Both properties have a common origin and appear in a number of phenomena.

The action of NC gauge theory resembles that of ordinary Yang–Mills theory, but with the star products in addition to ordinary matrix multiplication. The general form of the gauge-invariant action for gauge fields is [17]

$$S_{\mathrm{gauge}} = -\frac{1}{2}\int \mathrm{d}^4x \,\mathbf{Tr}\, \frac{1}{\mathbf{G}^2} \widehat{F}_{\mu\nu} \star \widehat{F}^{\mu\nu}. \tag{3}$$

Here $\mathbf{Tr}$ is a trace and $\mathbf{G}$ is an operator that encodes the coupling constants of the theory. Both will be discussed in detail below. The NC field strength is

$$\widehat{F}_{\mu\nu} = \partial_\mu \widehat{V}_\nu - \partial_\nu \widehat{V}_\mu - \mathrm{i}[\widehat{V}_\mu \stackrel{\star}{,} \widehat{V}_\nu] \tag{4}$$

and $\widehat{V}_\mu$ is the NC analog of the gauge vector potential. The Seiberg–Witten maps are used to express the non-commutative fields and parameters as functions of ordinary fields and parameters and their derivatives. This automatically ensures a restriction to the correct degrees of freedom. For the NC vector potential the SW map yields

$$\widehat{V}_\xi = V_\xi + \frac{1}{4}\theta^{\mu\nu}\{V_\nu, (\partial_\mu V_\xi + F_{\mu\xi})\} + \mathcal{O}\left(\theta^2\right), \tag{5}$$

where $F_{\mu\nu} \equiv \partial_\mu V_\nu - \partial_\nu V_\mu - \mathrm{i}[V_\mu, V_\nu]$ is the ordinary field strength and V_μ is the whole gauge potential for the gauge group $G_{\mathrm{SM}} \equiv SU(3)_C \times SU(2)_{\mathrm{L}} \times U(1)_Y$

$$V_\mu = g'\mathcal{A}_\mu(x)Y + g\sum_{a=1}^{3} B_{\mu,a}(x)T^a_{\mathrm{L}} + g_{\mathrm{s}}\sum_{b=1}^{8} G_{\mu,b}(x)T^b_{\mathrm{S}}. \tag{6}$$

It is important to realize that the choice of the representation in the definition of the trace $\mathbf{Tr}$ has a strong influence on the theory in the non-commutative case. The reason for this is that, owing to the Seiberg–Witten map, terms of higher than quadratic order in the Lie algebra generators will appear in the trace. The choice of the trace corresponds to a choice of the representation of the gauge group. The adjoint representation would not lead to new triple gauge boson interactions and, in particular, show no triple-photon vertices [17,18]. This, however, would be an ad hoc choice (unless we are discussing a GUT scenario). Let us emphasize again that the action that we present here should be understood as an effective theory.

From this point of view, all representations of gauge fields that appear in the SM have to be considered in the definition of the trace. Consequently, according to [17], we choose a trace over all particles with different quantum numbers in the model that have covariant derivatives acting on them. In the SM, these are, for each generation, five multiplets of fermions and one Higgs multiplet. The operator $\mathbf{G}$, which determines the coupling constants of the theory, must commute with all generators $(Y, T^a_{\mathrm{L}}, T^b_{\mathrm{S}})$ of the gauge group, so that it does not spoil the trace property of $\mathbf{Tr}$. This implies that $\mathbf{G}$ takes on constant values $g_1, \ldots, g_6$ on the six multiplets (Table 1 in [17]). The operator $\mathbf{G}$ is in general a function of Y and the Casimir operators of $SU(2)$ and $SU(3)$. However, because of the special assignment of hypercharges in the SM it is possible to express $\mathbf{G}$ solely in terms of Y.

The action up to linear order in θ allows new triple gauge boson interactions that are forbidden in the SM and has the following form:

$$\begin{aligned} S_{\mathrm{gauge}} &= -\frac{1}{4}\int \mathrm{d}^4x \, f_{\mu\nu} f^{\mu\nu} \\ &\quad -\frac{1}{2}\int \mathrm{d}^4x \,\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right) - \frac{1}{2}\int \mathrm{d}^4x \,\mathrm{Tr}\left(G_{\mu\nu}G^{\mu\nu}\right) \end{aligned} \tag{7}$$

$$
\begin{aligned}
&+g_{\mathrm{s}}\,\theta^{\rho\tau}\!\int \mathrm{d}^4x\,\mathrm{Tr}\left(\frac{1}{4}G_{\rho\tau}G_{\mu\nu} - G_{\mu\rho}G_{\nu\tau}\right)G^{\mu\nu}\\
&+{g'}^3\kappa_1\theta^{\rho\tau}\!\int \mathrm{d}^4x\,\left(\frac{1}{4}f_{\rho\tau}f_{\mu\nu} - f_{\mu\rho}f_{\nu\tau}\right)f^{\mu\nu}\\
&+g'g^2\kappa_2\,\theta^{\rho\tau}\!\int \mathrm{d}^4x\sum_{a=1}^{3}\left[\left(\frac{1}{4}f_{\rho\tau}F^a_{\mu\nu} - f_{\mu\rho}F^a_{\nu\tau}\right)F^{\mu\nu,a} + \text{c.p.}\right]\\
&+g'g_{\mathrm{s}}^2\kappa_3\,\theta^{\rho\tau}\!\int \mathrm{d}^4x\sum_{b=1}^{8}\left[\left(\frac{1}{4}f_{\rho\tau}G^b_{\mu\nu} - f_{\mu\rho}G^b_{\nu\tau}\right)G^{\mu\nu,b} + \text{c.p.}\right],
\end{aligned}
$$

where c.p. means cyclic permutations in f. Here $f_{\mu\nu}$, $F^a_{\mu\nu}$, and $G^b_{\mu\nu}$ are the physical field strengths corresponding to the groups $U(1)_Y$, $SU(2)_{\mathrm{L}}$, and $SU(3)_C$, respectively. The constants κ_1, κ_2, and κ_3 are parameters of the model. They are functions of $1/g_i^2$, $(i = 1, ..., 6)$ and have the following form:

$$
\begin{aligned}
\kappa_1 &= -\frac{1}{g_1^2} - \frac{1}{4g_2^2} + \frac{8}{9g_3^2} - \frac{1}{9g_4^2} + \frac{1}{36g_5^2} + \frac{1}{4g_6^2},\\
\kappa_2 &= -\frac{1}{4g_2^2} + \frac{1}{4g_5^2} + \frac{1}{4g_6^2},\\
\kappa_3 &= +\frac{1}{3g_3^2} - \frac{1}{6g_4^2} + \frac{1}{6g_5^2}.
\end{aligned}
\tag{8}
$$

In order to match the SM action at zeroth order in θ, three consistency conditions have been imposed in (7):

$$
\begin{aligned}
\frac{1}{{g'}^2} &= \frac{2}{g_1^2} + \frac{1}{g_2^2} + \frac{8}{3g_3^2} + \frac{2}{3g_4^2} + \frac{1}{3g_5^2} + \frac{1}{g_6^2},\\
\frac{1}{g^2} &= \frac{1}{g_2^2} + \frac{3}{g_5^2} + \frac{1}{g_6^2},\\
\frac{1}{g_{\mathrm{s}}^2} &= \frac{1}{g_3^2} + \frac{1}{g_4^2} + \frac{2}{g_5^2}.
\end{aligned}
\tag{9}
$$

These three conditions together with the requirement that $1/g_i^2 > 0$, define a three-dimensional pentahedron in the six-dimensional moduli space spanned by $1/g_1^2, ..., 1/g_6^2$ [1].

From the action (7) we extract the neutral triple gauge boson terms which are not present in the SM Lagrangian. In terms of the physical fields (A, Z, G) they are

$$
\begin{aligned}
\mathcal{L}_{\gamma\gamma\gamma} &= \frac{e}{4}\sin 2\theta_{\mathrm{W}}\,K_{\gamma\gamma\gamma}\theta^{\rho\tau}A^{\mu\nu}\left(A_{\mu\nu}A_{\rho\tau} - 4A_{\mu\rho}A_{\nu\tau}\right),\\
K_{\gamma\gamma\gamma} &= \frac{1}{2}\,gg'(\kappa_1 + 3\kappa_2);
\end{aligned}
\tag{10}
$$

$$
\begin{aligned}
\mathcal{L}_{Z\gamma\gamma} &= \frac{e}{4}\sin 2\theta_{\mathrm{W}}\,K_{Z\gamma\gamma}\,\theta^{\rho\tau}\left[2Z^{\mu\nu}\left(2A_{\mu\rho}A_{\nu\tau} - A_{\mu\nu}A_{\rho\tau}\right)\right.\\
&\quad \left. + 8Z_{\mu\rho}A^{\mu\nu}A_{\nu\tau} - Z_{\rho\tau}A_{\mu\nu}A^{\mu\nu}\right],\\
K_{Z\gamma\gamma} &= \frac{1}{2}\left[{g'}^2\kappa_1 + \left({g'}^2 - 2g^2\right)\kappa_2\right];
\end{aligned}
\tag{11}
$$

$$
\begin{aligned}
\mathcal{L}_{ZZ\gamma} &= \mathcal{L}_{Z\gamma\gamma}(A \leftrightarrow Z),\\
K_{ZZ\gamma} &= \frac{-1}{2gg'}\left[{g'}^4\kappa_1 + g^2\left(g^2 - 2{g'}^2\right)\kappa_2\right];
\end{aligned}
\tag{12}
$$

$$
\begin{aligned}
\mathcal{L}_{ZZZ} &= \mathcal{L}_{\gamma\gamma\gamma}(A \to Z),\\
K_{ZZZ} &= \frac{-1}{2g^2}\left[{g'}^4\kappa_1 + 3g^4\kappa_2\right];
\end{aligned}
\tag{13}
$$

$$
\begin{aligned}
\mathcal{L}_{Zgg} &= \mathcal{L}_{Z\gamma\gamma}(A \to G^b),\\
K_{Zgg} &= \frac{g_{\mathrm{s}}^2}{2}\left[1 + \left(\frac{g'}{g}\right)^2\right]\kappa_3;
\end{aligned}
\tag{14}
$$

$$
\begin{aligned}
\mathcal{L}_{\gamma gg} &= \mathcal{L}_{Zgg}(Z \to A),\\
K_{\gamma gg} &= \frac{-g_{\mathrm{s}}^2}{2}\left[\frac{g}{g'} + \frac{g'}{g}\right]\kappa_3,
\end{aligned}
\tag{15}
$$

where $A_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$, et cetera.

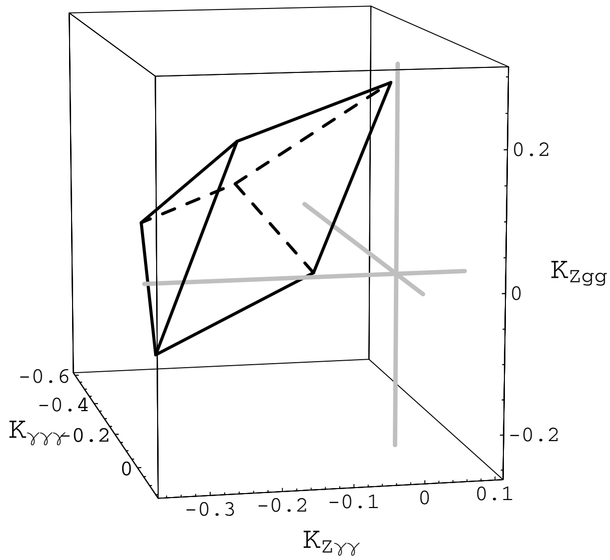

Fig. 1. The three-dimensional pentahedron that bounds possible values for the coupling constants $K_{\gamma\gamma\gamma}$, $K_{Z\gamma\gamma}$ and K_{Zgg} at the M_Z scale. The vertices of the pentahedron are $(-0.184, -0.333, 0.054)$, $(-0.027, -0.340, -0.108)$, $(0.129, -0.254, 0.217)$, $(-0.576, 0.010, -0.108)$, $(-0.497, -0.133, 0.054)$, and $(-0.419, 0.095, 0.217)$

Figure 1 shows the three-dimensional pentahedron that bounds allowed values for the dimensionless coupling constants $K_{\gamma\gamma\gamma}$, $K_{Z\gamma\gamma}$ and K_{Zgg}. For any chosen point within the pentahedron in Fig. 1 the remaining three coupling constants (12), (13) and (15), i.e. $K_{ZZ\gamma}$, K_{ZZZ} and $K_{\gamma gg}$ respectively, are uniquely fixed by the NCSM. This is true for any combination of three coupling constants from (10)–(15).

Experimental evidence for non-commutativity coming from the gauge sector should be searched for in processes which involve the above vertices. The simplest and most natural choice are the $Z \to \gamma\gamma$, gg decays, allowed for real (on-shell) particles. All other simple processes, such as $\gamma \to \gamma\gamma$, gg, and $Z \to Z\gamma$, ZZ, are on-shell forbidden by kinematics. The $Z \to \gamma\gamma$, gg decays are strictly forbidden in the SM by angular momentum conservation

[1] In terms of the couplings g_i these are complicated equations describing a family of hyper ellipsoids, however, in terms of $1/g_i^2$ they form a set of linear equations

and Bose statistics (Yang theorem) [19,20]; therefore, they both could serve as a clear signal for the existence of spacetime non-commutativity[2].

The $Z \to \gamma\gamma$ process has a tiny SM background from the rare $Z \to \pi^0\gamma$, $\eta\gamma$ decays. At high energies, the two photons from the π^0 or η decay are too close to be separated and they are seen in the electromagnetic calorimeter as a single high energy photon [21]. The SM branching ratios for these rare decays are of order 10^{-11} to 10^{-10} [22]. This is much smaller than the experimental upper bounds which are of order 10^{-5} for the all three branching ratios ($Z \to \gamma\gamma$, $\pi^0\gamma$, $\eta\gamma$) [23]. The experimental upper bound, obtained from the $e^+e^- \to \gamma\gamma$ annihilation, for $\Gamma_{Z\to\gamma\gamma}$ is $< 1.3 \times 10^{-4}$ GeV [23].

The $Z \to gg$ decay mode should be observed in $Z \to$ 2 jets processes. However, it could be smothered by the strong $Z \to q\bar{q}$ background, i.e. by hadronization, which also contains NC contributions. Since the hadronic width of the Z is in good agreement with the QCD corrected SM, the $Z \to gg$ can at most be a few percent. Taking into account the discrepancy between the experimentally observed hadronic width for the Z-boson and the theoretical estimate based on the radiatively corrected SM, we estimate the upper bound for any new hadronic mode, like $\Gamma_{Z\to gg}$, to be $\sim 10^{-3}$ GeV [23].

We now derive the partial widths for the $Z(p) \to \gamma(k)\,\gamma(k')$ decay. Care has to be taken when one tries to compute matrix elements in NCGFT. In our model, the *in* and *out* states can be taken to be ordinary *commutative* particles. Quantization is straightforward to the order in θ that we have considered; Feynman rules can be obtained either via the Hamiltonian formulation or directly from the Lagrangian; a rather convenient property of the action, relevant to computations, is its symmetry under ordinary gauge transformations in addition to noncommutative ones.

From the Lagrangian $\mathcal{L}_{Z\gamma\gamma}$, it is easy to write the gauge-invariant amplitude $\mathcal{M}_{Z\to\gamma\gamma}$ in momentum space. Since we are dealing with a SM forbidden process, this is essentially done using a distorted wave Born approximation. It gives

$$\sum_{\text{spins}} |\mathcal{M}_{Z\to\gamma\gamma}|^2 = -\theta^2 + \frac{8}{M_Z^2}(p\theta^2 p) - \frac{16}{M_Z^4}(k\theta k')^2 \,. \quad (16)$$

From above equation and in the Z-boson rest frame, the partial width of the $Z \to \gamma\gamma$ decay is

$$\Gamma_{Z\to\gamma\gamma} = \frac{\alpha}{12} M_Z^5 \sin^2 2\theta_{\mathrm{W}} K_{Z\gamma\gamma}^2 \left[\frac{7}{3}(\vec{\theta}_{\mathrm{T}})^2 + (\vec{\theta}_{\mathrm{S}})^2\right], \quad (17)$$

where $\vec{\theta}_{\mathrm{T}} = \{\theta^{01}, \theta^{02}, \theta^{03}\}$ and $\vec{\theta}_{\mathrm{S}} = \{\theta^{23}, \theta^{13}, \theta^{12}\}$, are responsible for time-space and space–space non-commutativity, respectively. This result differs essentially from that given in [14] where the $\Gamma_{Z\to\gamma\gamma}$ partial width depends only on time-space non-commutativity.

[2] The Z and γ self-couplings vanish identically in the SM if all particle are on-shell. They can, however, appear if one of the photons is considered an off-shell particle in the s-channel [20]

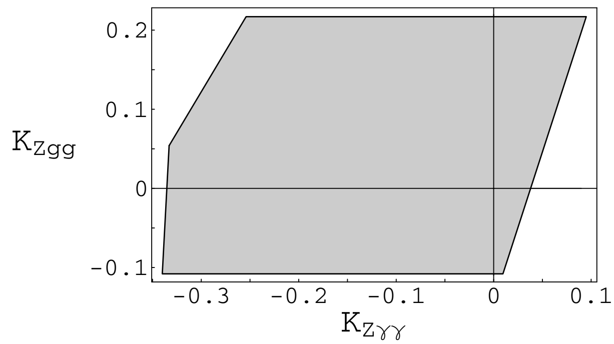

Fig. 2. The allowed region for $K_{Z\gamma\gamma}$ and K_{Zgg} at the M_Z scale, projected from the pentahedron given in Fig. 1. Note that $K_{\gamma\gamma\gamma}$ is non-zero at the point where both $K_{Z\gamma\gamma}$ and K_{Zgg} vanish. The vertices of the polygon are $(-0.254, 0.217)$, $(-0.333, 0.054)$, $(-0.340, -0.108)$, $(0.010, -0.108)$ and $(0.095, 0.217)$

For the Z-boson at rest and polarized in the direction of the 3-axis, we find that the *polarized* partial width is

$$\Gamma_{Z^3\to\gamma\gamma} = \frac{\alpha}{4} M_Z^5 \sin^2 2\theta_{\mathrm{W}} K_{Z\gamma\gamma}^2 \times \left[\frac{2}{5}\left((\theta^{01})^2 + (\theta^{02})^2\right) + \frac{23}{15}(\theta^{03})^2 + (\theta^{12})^2\right]. \quad (18)$$

In the absence of time-space non-commutativity a sophisticated, sensibly arranged polarization experiment could in principal determine the vector of $\vec{\theta}_{\mathrm{S}}$. The NC structure of spacetime may depend on the matter that is present. In our case it is conceivable that the direction of $\vec{\theta}_{\mathrm{T,S}}$ may be influenced by the polarization of the Z particle. In this case, our result for the *polarized* partial width is particularly relevant.

Due to the same Lorentz structure of the Lagrangians $\mathcal{L}_{Z\gamma\gamma}$ and $\mathcal{L}_{Zgg}$ we find

$$\frac{\Gamma_{Z\to gg}}{\Gamma_{Z\to\gamma\gamma}} = \frac{\Gamma_{Z^3\to gg}}{\Gamma_{Z^3\to\gamma\gamma}} = 8\frac{K_{Zgg}^2}{K_{Z\gamma\gamma}^2}. \quad (19)$$

The factor of eight in the above ratios is due to color.

In order to estimate the NC parameter from upper bounds $\Gamma^{\mathrm{exp}}_{Z\to\gamma\gamma} < 1.3\times10^{-4}$ GeV and $\Gamma^{\mathrm{exp}}_{Z\to gg} < 1\times10^{-3}$ GeV [23] it is necessary to determine the range of couplings $K_{Z\gamma\gamma}$ and K_{Zgg}. The allowed region for the coupling constants $K_{Z\gamma\gamma}$ and K_{Zgg} is given in Fig. 2. Since $K_{Z\gamma\gamma}$ and K_{Zgg} could be zero simultaneously it is not possible to extract an upper bound on θ only from the above experimental upper bounds alone.

We need to consider an extra interaction from the NCSM gauge sector, namely the triple-photon vertex, to estimate θ. The important point is that the triplet of coupling constants $K_{\gamma\gamma\gamma}$, $K_{Z\gamma\gamma}$ and K_{Zgg}, as well as the pair of couplings $K_{\gamma\gamma\gamma}$ and $K_{Z\gamma\gamma}$ *can never vanish simultaneously* due to the constraint set by the value of the SM coupling constants at the weak interaction scale. This can be seen from the pentahedron in Fig. 1. In conclusion, it is possible to estimate θ from the NCSM gauge sector

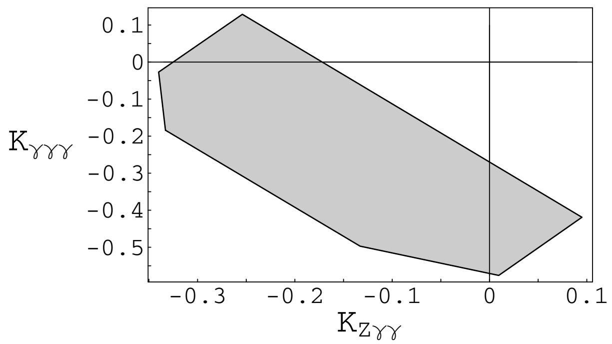

Fig. 3. The allowed region for $K_{Z\gamma\gamma}$ and $K_{\gamma\gamma\gamma}$ at the M_Z scale, projected from the pentahedron given in Fig. 1. The vertices of the polygon are $(-0.333, -0.184)$, $(-0.340, -0.027)$, $(-0.254, 0.129)$, $(0.095, -0.419)$, $(0.0095, -0.576)$, and $(-0.133, -0.497)$

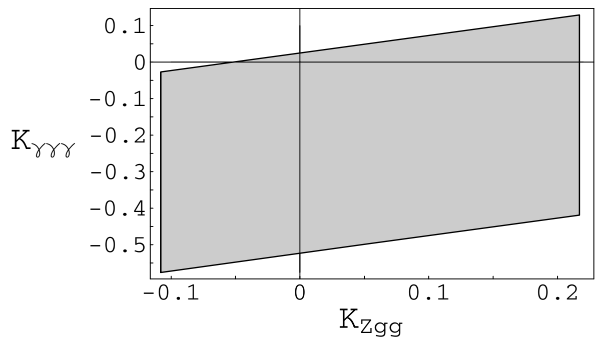

Fig. 4. The allowed region for $K_{\gamma\gamma\gamma}$ and K_{Zgg} at the M_Z scale, projected from the pentahedron given in Fig. 1. Note that $K_{Z\gamma\gamma}$ is non-zero at the point where both $K_{\gamma\gamma\gamma}$ and K_{Zgg} vanish. The vertices of the polygon are $(-0.108, -0.576)$, $(-0.108, -0.027)$, $(0.217, 0.129)$, $(0.217, -0.419)$, and $(0.054, -0.497)$

through a combination of various types of processes containing the $\gamma\gamma\gamma$ and $Z\gamma\gamma$ vertices. These are processes of the type $2 \to 2$, such as $e^+e^- \to \gamma\gamma$, $e\gamma \to e\gamma$, and $\gamma\gamma \to e^+e^-$ in leading order. The analysis has to be carried out in the same way as in [12]. Theoretically consistent modifications of relevant vertices are, however, necessary. Finally, we present the allowed region for the pair of couplings $K_{\gamma\gamma\gamma}$ and K_{Zgg} in Fig. 4. Note that Figs. 2–4 represent projections of pairs of coupling constants from the three-dimensional pentahedron spanned by the constants $K_{\gamma\gamma\gamma}$, $K_{Z\gamma\gamma}$ and K_{Zgg}.

The structure of our main results (16) to (19) remains the same for $SU(5)$ and $SU(3)_C \times SU(3)_\mathrm{L} \times SU(3)_\mathrm{R}$ GUTs that embed the NCSM that is based on the SW map [24, 18]; only the coupling constants change. Note that in the particular case of an $SO(10)$ GUT there is no triple gauge boson coupling [18].

In this article we have proposed two SM strictly forbidden decay modes, namely, $Z \to \gamma\gamma, gg$, as a possible signature of the NCSM. The experimental discovery of $Z \to \gamma\gamma, gg$ decays would certainly indicate a violation of the presently accepted SM and the definitive appearance of new physics.

In conclusion, the gauge sector of the non-minimal NCSM is an excellent place to discover spacetime non-commutativity experimentally [25–29], but not the best place to find bounds that exclude it. We hope that the importance of a possible discovery of non-commutativity of spacetime will convince experimentalists to look for SM forbidden decays in the gauge sector. A good reason for this is that the sensitivity to the non-commutative parameter $\theta^{\mu\nu}$ could be in the range of the next generation of linear colliders with a c.m.e. around a few TeV.

Acknowledgements. We would like to thank P. Aschieri, B. Jurčo and H. Štefančić for helpful discussions. One of us (NGD) would like to thank the University of Hawaii Theory Group for hospitality. This work was supported by the Ministry of Science and Technology of the Republic of Croatia under Contract No. 0098002, and by the US Department of Energy, Grant No. DE-FG06-85ER 40224.

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